MIXING JUICE TASK
LESSON GUIDE

LESSON OVERVIEW

In the *Mixing Juice* task, students encounter an open-ended problem where they are asked to compare the "orangeyness" of four drink mixes. Students will likely approach the task using a range of different strategies, making comparisons among the mixes with ratios, percents, and fractions. Students will investigate how ratios can be formed and scaled up to find equivalent ratios. In addition, students will use proportional reasoning to decide how to use the different mixes to make juice for 240 people.

The strategies for comparing the mixes will be compared and connected during the whole-group discussion. Students should be able to see how each form, ratios, percents, and fraction, provides information needed to derive one of the other forms.

Before working on the *Mixing Juice* task, the Warm-Up task *Comparing by Using Ratios* will focus students' attention on different ways to form ratios and different notations for ratios. Allow 15 to 20 minutes for students to engage in and discuss the Warm-Up task. The *Mixing Juice* task is a challenging problem for students and at least 1.5 class periods should be allowed for students to explore and have a whole-class discussion on the task.

COMMON CORE STATE STANDARDS

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
    c) Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity).
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.
- **7.RP.3** Use proportional relationships to solve multistep ratio and percent problems. *Example: percent increase*.

<table>
<tr>
<td>

ESSENTIAL QUESTIONS:

- What are different types of ratios and how are ratios used to make comparisons?
- What strategies can be used to compare ratios?
- How are ratios related to fractions?

</td>
<td>

NCTM ESSENTIAL UNDERSTANDINGS[2]:

5. Reasoning with ratios involves attending to and coordinating two quantities.
6. A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.
7. Forming a ratio as a measure of a real-world attribute involves isolating that attribute from other attributes and understanding the effect of changing each quantity on the attribute of interest.
8. A number of mathematical connections link ratios and fractions:
    - Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning.
    - Ratios are often used to make “part-part” comparisons, but fractions are not.
    - Ratios can often be meaningfully reinterpreted as fractions.
9. Ratios can be meaningfully reinterpreted as quotients.
7. Proportional reasoning is complex and involves understanding that:
    - Equivalent ratios can be created by iterating and/or partitioning a composed unit;
    - If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.

</td>
<td>

SKILLS DEVELOPED:

- Use different representations to form ratios and make comparisons with ratios.
- Use visual and numerical strategies to compare ratios.
- Form equivalent ratios and use equivalent ratios to solve problems.

</td>
</tr>
<tr>
<td>

MATERIALS:
Warm-Up task *Comparing by Using Ratios* and *Mixing Juice* task sheet, Calculators, Chart paper.

</td>
<td colspan="2">

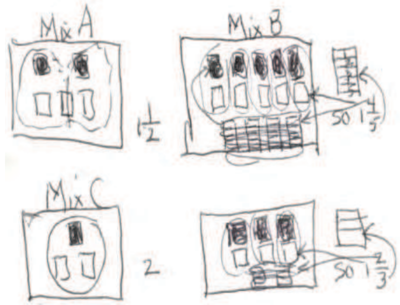
GROUPING:
Students will begin their work individually, but will then work in groups of three or four to discuss the task and arrive at a common solution.

</td>
</tr>
</table>

[2] NCTM (2010) *Developing Essential Understandings of Ratios, Proportions & Proportional Reasoning: Grades 6 -8.*

Warm-Up Task (*Comparing by Using Ratios)*	
SET-UP	
Tell students that a useful way to compare numbers is to form ratios. With students working in groups of three, give them the *Comparing by Using Ratios* task sheet and ask them to take turns reading the ratio statements to each other. Tell them they should form and interpret the ratios and think about different ways ratios can be written. They should also look for similarities and differences in the ratio statements.	
EXPLORE PHASE	
Monitoring Student Work: Circulate while students are reading and interpreting the ratios. Focus students' attention on the ratio comparisons. Listen to and make note of students' debating and deciding the types of ratios in Statements A-G. Tell students you will want them to share their reasons during the whole-class discussion.	
Possible Solution Paths Each ratio is a part-to-part ratio, a part-to-whole ratio, or a ratio comparing different kinds of measures or counts (also called a rate). Statement D compares a part to a whole. Statements C and F compare two different kinds of measures; this type of ratio is called a *rate.* The remaining statements compare parts to parts. Note that statement E can be interpreted as part-to-part or part-to-whole. Make note of students who are arguing either interpretation and highlight this during the Share Discuss Analyze Phase of the Warm Up. Ratios are often written in the form 5:6 or 5 to 6 to help students separate the ideas of ratios from fraction arithmetic.	• *How are these statements similar?* • *How are they different?*
SHARE DISCUSS ANALYZE PHASE	
General Considerations: Start by prompting students to focus on ratio statements that they think are similar, and ask them to explain why. You might start by asking them, "How are statements A and B similar?" Once they have identified part-part, you might then ask them if D is also part-part, thus distinguishing part-whole and part-part. You might then have them decide whether G fits one of these classes. Next, ask whether C and F fit into either of these groups. Name the groups as they are formed. You could then have a discussion of E – saying that you don't know which it is. Through this discussion you should separate the statements into the three types, and introduce the terminology – part-part, part-whole, and rate. If you allow 7-10 min for small-group discussion, that will leave only 10-13 minutes for the Share, Discuss, Analyze Phase.	

SET-UP (Mixing Juice Task)
Make sure students understand the context. Suggested Questions: • *How many of you have made juice by adding water to a mix before?* • *What was involved in making it?* You may want to bring in a can of frozen orange juice (thawed) and, with your class, make juice following the instructions on the can. You can discuss the fact that you have one container of concentrated juice and to this you add three containers of water (or whatever it says on the container of concentrate). Point out that the recipes given in the problem are different from the one on the can. At camp, the juice concentrate comes in a very large container without mixing proportions given. You might let students begin to explore the juice recipes C in small-group, and then reassemble in whole-group to discuss the groups' initial ideas about the different mixes. This approach gives groups a chance to consider several representations and comparison strategies. You might discuss parts A, B and C in whole-group and then challenge groups to solve part D. Make sure that students have solved D for at least two mixes before calling them together to discuss it C in whole-group. Remind students that they will be expected to: justify their solutions; explain their thinking and reasoning to others; make sense of other students' explanations; ask questions of the teacher or other students when they do not understand; and use correct mathematical language, and symbols.
EXPLORE PHASE: Supporting Students' Exploration of the Mathematical Ideas
<u>Private Think Time</u>: Allow students to work individually for 3 – 5 minutes without intervening, though you might want to circulate quickly to get an idea of the strategies that they are using. **<u>Small-Group Work</u>:** After 3 – 5 minutes, ask students to work with their partner or in their small groups. As students are working, circulate around the room. Be persistent in: • asking questions related to the mathematical ideas, problem-solving strategies, and connections between representations. • asking students to explain their thinking and reasoning. • asking students to explain in their own words, and build onto, what other students have said. As you circulate, identify solution paths that you will have groups share during the Share, Discuss, Analyze Phase, and decide on the sequence that you would like for them to be shared. Give groups a "heads up" that you will be asking them to put their solutions on chart paper to share during the whole-class discussion. Having the various strategies on chart paper will allow you to arrange the work in the room in a way that supports analyzing and making connections between and among them. Some students will start with naïve strategies such as simply finding the difference between the number of cans of concentrate and ignore the water. Challenge this idea by asking: *Can I keep adding cans of water without making the juice less orangey?* Questions A and B will allow misconceptions (additive strategies) as well as alternate multiplicative approaches for comparing ratios to emerge. Question C is designed to raise the issue that the phrase "of Mix B" signals that this is a part-whole statement, thus 5/9 is not correct. Though students will discuss it during the Explore Phase, you will focus on questions A, B and D as you circulate during the Explore Phase.

<table>
<tr><th colspan="2">Possible Solution Paths for Parts A and B</th><th>Possible Assessing and Advancing Questions</th></tr>
<tr>
<td>
</td>
<td>Draw pictures to show how much water there is for each cup of concentrate in each mix. The goal is to partition the water squares so that each cup of concentrate gets the same amount of water. In this way, you can see that Mix C has the most water for each cup of concentrate (least orangey) and Mix A has the least amount of water (most orangey).</td>
<td>Assessing Questions

Tell me about your diagram. What does it show?
How does this help to decide which mix is most or least orangey?
What kind of ratios does this visual represent?

Advancing Questions

If you wanted to write numerical ratios to represent what you have in this visual strategy, what would they look like? How would they help you decide which is most or least orangey?
</td>
</tr>
<tr>
<td><table>
<tr><td>Mix
C
$\frac{1}{3}$</td><td>Mix
B
$\frac{5}{14}$</td><td>Mix
D
$\frac{3}{8}$</td><td>Mix
A
$\frac{2}{5}$</td></tr>
</table></td>
<td>Use part-to-whole ratios written in fraction form to express the relationships of concentrate to total liquid in a batch.

Using prior knowledge about fractions, students may represent the fractions as decimals or percents. There are a variety of strategies that can then be used to order the fractions, i.e., benchmark comparisons or common denominators.</td>
<td>Assessing Questions

Tell me about your work.
Tell me, for example, what $\frac{3}{8}$ means in the context of the problem. What does the 3 represent? The 8? Is there any way you can let me know that in your explanation?

Advancing Questions (if no comparisons are being made)

How are you going to compare the mixes? Which is most orangey and how do you know?
What are some ways that you have used in the past to make comparisons? See if you can think about some of these and then try to make comparisons, in one or two ways.

Advancing Questions (if comparisons are being made)

How does knowing that $\frac{1}{3}$ is the ratio of concentrate to mix help you to answer Question D?
</td>
</tr>
<tr>
<td>A. $\frac{1\frac{1}{2}}{1}; \frac{1\frac{4}{5}}{1}; \frac{2}{1}; \frac{1\frac{2}{3}}{1}$

B. $\frac{\frac{2}{3}}{1}; \frac{\frac{5}{9}}{1}; \frac{\frac{1}{2}}{1}; \frac{\frac{3}{5}}{1}$</td>
<td>A. Figure out how much water goes with each cup of concentrate. Notice that with these ratios we focus on most and least water.
B. Figure out how much concentrate goes with each cup of water. Notice that with these ratios we focus on most and least concentrate.</td>
<td>Assessing Questions

Tell me about your work. What did you do and why?
What kind of ratios have you created? What do they represent? Is there any way you can let me know that in your explanation?

Advancing Questions

How are you using your ratios to decide which is most orangey?
How are you going to compare the mixes? Which is most orangey and how do you know?
What strategy are you using to order your ratios? What does the order mean in the context of the problem?
</td>
</tr>
</table>

Mix A $\frac{2:3}{30:45}$ Mix B $\frac{5:9}{30:54}$ Mix C $\frac{1:2}{30:60}$ Mix D $\frac{3:5}{30:50}$	Use part-to-part ratios and make the number of cups of concentrate the same. Notice that with these ratios we focus on most and least water.	*Assessing Questions* • *Tell me about your work.* • *What kind of ratios have you created? What do they represent? Is there any way you can let me know that in your explanation?* • *How did you decide that 30 cups of concentrate would be a helpful amount?* • *How did you make the new ratios? What strategy did you use?* • *How does making the amount of concentrate the same help you to reason about the problem?* *Advancing Question* • *How are you going to compare the mixes? Which is most orangey and how do you know?* • *Tell me which is the orangeyist and how you know? Write an explanation.*

Possible Errors and Misconceptions for Parts A and B	Possible Questions to Address Errors and Misconceptions
Comparing the amount of water using absolute differences approach:  A. The juice that will taste most orangey is Mix C because it does not have as much water as mixes A, B, and D. B. The juice that will taste least orangey is Mix B because it has more water than mixes A, C, and D.	Assessing Question • *Tell me about your work. Explain your thinking.* *Advancing Questions* • *What would 2 batches of Mix C look like? How would the "orangeyness" of this new one compare to Mix A? Why?* • *For every one cup of water in Mix A how many cups of concentrate would I have?*

Possible Solution Paths for Part D	Possible Assessing and Advancing Questions
Find the number of batches needed to make 120 cups of juice from each recipe. Then multiply to find the amount of water and concentrate. For example, one batch of Mix A makes 5 cups of juice and since 120 cups are needed, 120 + 5 yields 24 batches. So, 2 x 24 or 48 cups of concentrate and 3 x 24 or 72 cups of cold water are needed. 48 cups of concentrate plus 72 cups of water yields 120 cups of juice for the 240 campers.	Assessing Questions • Why *did you decide to work with 120 cups of juice?* • *How did you use 120 to help you solve the problem?* • *What does the 24 mean in the context of the problem and why did you multiply the 2 and 3 by it?* Advancing Questions • *How can you check to see that the amounts you calculated are correct?* • *So the ratio of cups of juice in the big batch to cups of juice in the recipe is 120:5 (pointing to 120 + 5 on the paper). What is the ratio of cups of concentrate in the big batch to cups of concentrate in the recipe? What about the ratio of cups of water in the big batch to cups of water in the recipe? Do you think that will happen with the other mixes, too?* • *Write those ratios in equation form and study the way they look. Is there a way to decide by looking at the statement that two ratios are equal*

<table>
<tr>
<td>
Make a rate or ratio table to scale up:
Mix D (for example)

Concentrate in cups	Water in cups	Total in cups
3	5	8
6	10	16
9	15	24
12	20	32
15	25	40
45	75	120

</td>
<td>
Students may continue to add 3:5:8 to each row until they reach 45:75:120, or they may notice that they can multiply a row by a scale factor to get to their result more quickly as is shown in the last 2 rows of this table.
</td>
<td>
Assessing Questions

- *Why did you decide to organize your work in a table? How is that helpful?*
- *How are you getting from one row to the next in your table?*
- *How did you know when to stop making new rows?*
- *What patterns do you see in the table?*

Advancing Questions

- *Can you show what you did in the last two rows so we don't have to guess? Use either an numerical expression or a written explanation. Then say WHY you knew you could do what you did.*

</td>
</tr>
</table>

Possible Errors and Misconceptions for Part D	Possible Questions to Address Errors and Misconceptions
If students are stuck on the question of making a recipe for 240 people, ask them to consider Mix A to start.	• *How much total juice does one batch of Mix A make? How can we figure out how many people one batch of this juice will serve?* • *What if each serving is one cup? What if each serving is ½ cup?* • *If you were going to serve juice to 50 people, how many cups of juice would you have to make if each person gets ½ cup of juice? How many batches of juice would this be?* • *What are different strategies you might use to answer this question?* (Students might divide 50 people by 10 servings per batch to determine that 5 batches are needed. Alternatively, some students may reason that if 1 batch makes 10 servings, then 2 batches make 20 servings, 3 batches makes 30 servings, etc. Students may make a table from which to reason.

Servings of Juice					
Number of Batches	1	2	3	4	5
Servings	10	20	30	40	50

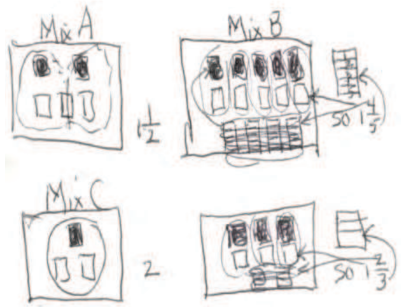
SHARE DISCUSS ANALYZE PHASE: Deepening and Connecting Mathematical Understanding

General Considerations:

- Orchestrate the class discussion so that it builds on, extends, and connects the thinking and reasoning of students.
- Sequence the solution paths so that you will be able to press students to make comparisons and connections across, and between, the various solution paths.

It is recommended that you have groups share and discuss Parts A and B, then discuss C and D. For Parts A and B, the point you want students to think about is: *What does it mean to be most orangey tasting? To be least orangey tasting? Why do we have to consider BOTH the amount of concentrate AND the amount of water?*

Possible Sequence of Solution Paths	Possible Questions and *Possible Student Responses*
Begin with the visual strategy under Possible Misconceptions and Errors for Parts A and B.	While the reasoning that was used to decide which mix is most orangey is faulty, the visual can be used to make connections to some of the other strategies. You might decide to have the group share their drawing and come back to them after they have heard some of the other strategies described to see if they will reconsider their reasoning and their answer about which is most orangey.
Have a group who used a part-to-whole ratio share their strategy. If there are a variety of strategies that were used to reason about the part-to-whole ratios, start with reasoning from the ratio itself, then move to those that converted the ratio to a decimal, then move to percents, and finally connect to the common denominator approach.	**How did your group decide to compare the mixes?** • *We made our comparisons using part-to-whole ratios. We compared the part of the mix that is concentrate to the whole mix. Then we made these fractions into decimals and looked for the largest because that would tell us which mix was most orangey.* **How is your strategy related to the diagrams we saw in the first strategy?** • *The numerator in our fraction is the number of pieces that are shaded and these represent the concentrate. The denominator in our fraction is the total number of pieces in the diagram for a particular mix.* **How do your decimals relate to the diagrams?** • *The decimal (percent) would represent the portion of the whole diagram that is shaded. In this case, the whole mix would have a value of 1 or 100%.*

Use a visual that shows how much water there is for each cup of concentrate in each mix. This is a unit rate approach. 	**Tell us what your drawing means.** • *The solid squares represent cups of concentrate and the empty squares represent cups of water. We divided the water squares so that each cup of concentrate gets the same amount of water.* **How does your drawing help you decide which mix is most orangey?** • *We can see that Mix A has the least water, 1 ½ cups, for each cup of concentrate, so it is the most orangey.* **How does your drawing help you decide which mix is least orangey?** • *We can see that Mix C has the most water, 2 cups, for each cup of concentrate, so it is the least orangey* **How does your strategy compare to the ones we saw earlier?** • *We were using part-to-part comparisons and they were using part-to-whole comparisons.* • *For them to say which mix was most orangey they had to look for the mix that had the most concentrate. For us to say which mix was most orangey, we looked for the mix that has the least water.*
Have a group that used a unit rate approach with unit ratios expressed numerically share next.	**Tell us about the ratios your group made. How did you calculate them?** • *We compared cups of water to cups of concentrate. We wanted to figure out how much water goes with each cup of concentrate, so we divided the number of cups of water by the number of cups of concentrate.* **How does this compare to the last groups' strategy?** • *They divided up the area of the squares and we used ratios, unit rates, but they both convey similar information. Like for Mix B, our ratio was* $\frac{1\frac{4}{5}}{1}$ *which is water to concentrate and that's what their picture shows.*
Have a group that used part-to-part ratios and made the number of cups of concentrate (or water) the same. *(This strategy is helpful to scaffold student thinking for Part D.)*	**Tell us about your approach. What kind of ratios did you use to compare the mixes?** • *We used part-to-part ratios. We thought if we had 4 big pots and used each pot to make many batches of each mix we could compare them. We wanted each pot to have the same amount of concentrate so we could think about how much water is in each pot.* **How will the multiple batches of juice in the pots compare to the original mix?** • *The juice in each pot will taste the same as the original mix because we kept the ratio of concentrate to water the same.* **How does your strategy compare to the other strategies we have seen?** • *Our strategy is probably most like the part-to-whole ratios where they found common denominators. Even though we used part-to-part ratios to reason, their strategy was similar because, like us, they ended up with a lot more juice than was in the original mix.*

CLOSURE
Quickwrite: • Why is a ratio a useful way to make comparisons? • Which of the following will taste the most orangey? 2 cups of concentrate and 3 cups of water; 4 cups of concentrate and 6 cups of water; or 10 cups of concentrate and 15 cups of water? Explain your reasoning.
Possible Assessment: • Provide some additional contexts where they need to compare quantities. Ask them to explain their thinking in writing.
Homework: • Find items from the current curriculum that will allow them to apply these ideas and understandings.

References

Lappan, Fey, Fitzgerald, Friel, Phillips (2009). Teacher's Guide: Connected Mathematics 2. *Comparing and Scaling: Ratio, Proportion, and Percent,* Pearson.

Comparing by Using Ratios

A useful way to compare numbers is to form ***ratios***. Talk to your classmates about what is the same and what is different about these ratio statements.

A. In taste tests, people who preferred Bolda Cola outnumbered those who preferred Cola Nola by a ratio of 3 to 2.

B. The ratio of boys to girls in our class is 12 boys to 15 girls.

C. For every four tents there are 12 scouts.

D. The ratio of boys to students in our class is 12 boys to 27 students.

E. The ratio of kittens to cats in our neighborhood is $\frac{1}{4}$.

F. The sign in the hotel lobby says:

1 dollar Canadian : 0.85 dollars U.S.

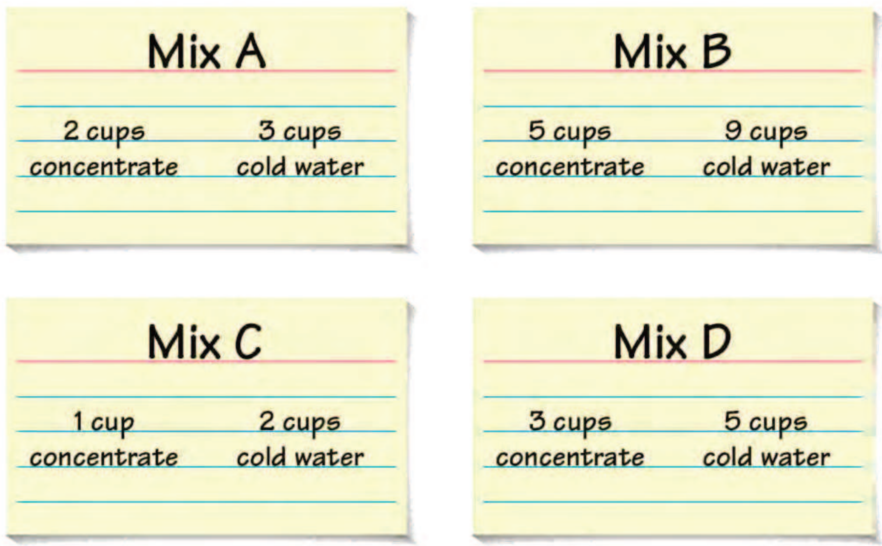
G. A paint mixture calls for 5 parts blue paint to 2 parts yellow paint.

From Lappan et al. (2002). Comparing and Scaling: Ratio, Proportion, Percent, p. 26. Connected Mathematics, Pearson.

MIXING JUICE

Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times.

One morning, Julia and Mariah make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange juice concentrate. To find the mix that tastes best, they decide to test some mixes.



Developing Comparison Strategies

A. Which mix will make juice that is the most "orangey"? Explain.

B. Which mix will make juice that is the least "orangey"? Explain.

C. Which comparison statement is correct? Explain.

a. $\frac{5}{9}$ of Mix B is concentrate $\frac{5}{14}$ of Mix B is concentrate

D. Assume that each camper will get $\frac{1}{2}$ cup of juice.

1. For each mix, how many batches are needed to make juice for 240 campers?
2. For each mix, how much concentrate and how much water are needed to make juice for 240 campers?

From Lappan et al. (2009). *Comparing and Scaling: Ratio, Proportion, Percent*, p. 19, Lesson 2.1. Connected Mathematics 2, Pearson.

OUNCES OF COFFEE TASK
SEVENTH GRADE LESSON GUIDE

LESSON OVERVIEW:

Students will be presented with the Ounces of Coffee problem. They will be asked to determine whether or not the there is a proportional relationship between the ounces of coffee to the price. Students then will be asked to find the unit price and explain in writing what the unit price means in the context of the problem. Finally, students will explain why it is helpful to determine if the relationship between the amount of coffee and price is proportional.

Students should be able to see the proportional relationship between the ounces of coffee and the price. They should be able to find the unit price and to see that the cost per ounce is the same.

Before working on the Ounces of Coffee task, students will do a Warm-Up task identifying ratios and the appropriate representations of unit rates. Students will be allowed 15-20 minutes to engage in and discuss the Warm-Up task. At least 2-3 periods will be allotted for students to explore and share their work.

COMMON CORE STATE STANDARDS:

- **6.RP.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
    a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
    b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
    c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
    d. Use ratio reasoning to convert measurement units to manipulate and transform units appropriately when multiplying or dividing quantities.
- **7.RP.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*
- **7.RP.2** Recognize and represent proportional relationships between quantities.
    d. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
    e. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.